TURBULENCE IN CALCULATIONS OF HEAT TRANSFER AT THE STAGNATION POINT OF A JET INTERACTING NORMALLY WITH A PLANAR OBSTACLE

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Calculations of heat transfer at the stagnation point of subsonic and supersonic axisymmetric jets interacting with a normally positioned planar obstacle are performed. Results of the calculations are compared with available experimental data.

The interaction of subsonic and supersonic jets with obstacles situated along the normal to the jet axis is characterized by considerable heat fluxes going to the obstacle, and several times in excess of those calculated on the basis of familiar formulas which define q_W near the stagnation point. Experimental investigations have shown that discrepancies in the data of theory and experiment are accounted for by the influence of appreciably intense turbulent pulsations inherent in the jets on flow and on heat transfer [1-3]. As the jet impinges on the obstacle, the pulsations give rise to additional heat transfer and momentum. The increment in momentum can be characterized by the introduction of additional viscosity ν_ε due to the presence of turbulent pulsations in the jet. Since heat transfer and momentum transfer through the stream turbulence at the stagnation point takes place in the direction perpendicular to the wall, we assume that ν_ε is proportional to the intensity of the turbulence ε of the oncoming jet and to the distance y from the obstacle:

$$\nu_\varepsilon = K\varepsilon U_\infty y, \tag{1}$$

where U_∞ is the time-averaged velocity of the impinging stream (in the case of subsonic jets $U_\infty = U_a$, the velocity in the exit section of the nozzle; in the case of supersonic jets $U_\infty = U_1$, the velocity downstream of the shock appearing in front of the obstacle); K is a proportionality factor to be determined. This assumption was introduced for the first time in [4], in a discussion of an experimental investigation of the effect of induced turbulent pulsations of free flow around a cylinder on heat transfer at the stagnation point.

In this case, where the liquid is assumed to be incompressible, a self-similar solution of the Navier – Stokes system exists in the neighborhood of the stagnation point.

Using the equations for the velocity components v_r and v_y in the radial direction and along the normal to the obstacle, respectively,

$$v_r = \beta r f'(\eta), \; v_y = -2\sqrt{\nu\beta} f(\eta) \tag{2}$$

and for the temperature

$$\Theta(\eta) = \frac{T_w - T(\eta)}{T_w - T_\infty}, \tag{3}$$

where

$$\eta = y\sqrt{\frac{\beta}{\nu}},$$

and, introducing these expressions into the Navier– Stokes equations, we obtain the following system of equations

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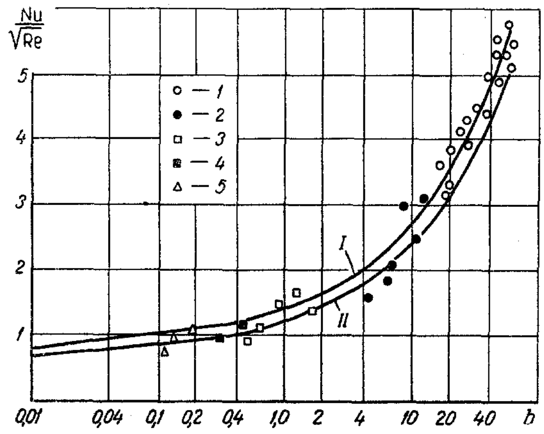

Fig. 1. Dependence of $\mathrm{Nu}/\sqrt{\mathrm{Re}}$ on parameter b characterizing turbulence of jet: I, II) calculations based on Eq. (7) at Pr = 1.0 and 0.74, respectively; 1) experiment staged at $M_a = 3.0$, n = 1.5 – 3.0, $y_0 = (0.5 - 4.0)d_a$, $\varkappa = 1.25$ [1, 2]; 2) experiment staged at $M_a = 0.785$, $y_0 = (7 - 10)d_a$, $\varkappa = 1.4$ [3]; 3) experiment staged at $M_a = 0.05$, $y_0 = (2 - 6)d_a$, $\varkappa = 1.4$ [5]; 4) experiment staged at $M_a = 0.05$; 0.2; 0.4, $y_0 = (2 - 6)d_a$, $\varkappa = 1.4$ [8]; 5) experiment staged at $M_a = 0.025 - 0.05$, $y_0 = (2 - 6)d_a$, $\varkappa = 1.4$ [7].

$$f'^2 - (2f + b) f'' = 1 + (1 + b\eta) f''', \tag{4}$$

$$\left(b\eta + \frac{1}{\mathrm{Pr}}\right)\Theta'' + (2f + b)\,\Theta' = 0, \tag{5}$$

where

$$b = K\varepsilon U_\infty \sqrt{\frac{1}{\nu\beta}}.$$

The boundary conditions for the solution (4) and (5) assume the form

$$f(0) = f'(0) = \Theta(0) = 0; \quad f'(\infty) = \Theta(\infty) = 1. \tag{6}$$

Figure 1 shows the dependence of $\Theta'(0) = \mathrm{Nu}/\sqrt{\mathrm{Re}}$ on the parameter b for Pr = 0.74 and 1.0. In this range of variation in Pr, the theoretical curve is approximated fairly well by the formula

$$\Theta'(0) = 0.763\,(1 + 0.75b^{0.52})\,\mathrm{Pr}^{0.4}$$

or

$$\mathrm{Nu} = 0.763\,(1 + 0.75b^{0.52})\,\mathrm{Pr}^{0.4}\sqrt{\mathrm{Re}}, \tag{7}$$

where

$$\mathrm{Nu} = \frac{\alpha d}{\lambda_w}; \quad \mathrm{Re} = \frac{\beta d^2}{\nu_w};$$

$d = d_a$; $\beta = (1.5U_a/d_a)(y_0/d_a)^{-0.22}$ in the case of subsonic jet over the initial interval [5]; $d = d_1$; $\beta = 2a_*/d_1$ in the case of a supersonic jet over the initial interval [1, 2].

Note that, when b = 0, Eq. (7) yields the familiar equation for Nu in the neighborhood of the stagnation point.

Taking the equation for b into acccunt, we convert Eq. (7) for a subsonic jet to the form

$$\mathrm{Nu} = 0.94\left(\frac{y_0}{d_a}\right)^{-0.11}\left[1 + 0.675K^{0.52}\varepsilon^{0.52}\left(\frac{y_0}{d_a}\right)^{0.06}\mathrm{Re}^{0.27}\right]\mathrm{Pr}^{0.4}\sqrt{\mathrm{Re}}, \tag{8}$$

where

$$\mathrm{Nu} = \frac{\alpha d_a}{\lambda_w}; \quad \mathrm{Re} = \frac{U_a d_a}{\nu_w},$$

and for a supersonic jet

$$\mathrm{Nu} = 1.08\,(1 + 0.6K^{0.52}\varepsilon^{0.52}M_*^{0.52}\mathrm{Re}^{0.27})\,\mathrm{Pr}^{0.4}\sqrt{\mathrm{Re}}, \tag{9}$$

where

$$\mathrm{Nu} = \frac{\alpha d_1}{\lambda_w}; \quad \mathrm{Re} = \frac{a_* d_1}{\nu_w}; \quad M_* = \frac{U_1}{a_*}.$$

The intensity of the turbulence ε needed in order to determine the proportionality factor K in the case of subsonic jets is assumed to be roughly equal to the average intensity of a free jet in the cross section to the state where the nozzle exit section is removed from the obstacle; the value of ε was determined from experimental data cited in [6]. In the case of supersonic jets, ε was assumed equal to the average intensity of the jet turbulence in the neighborhood of the stagnation point downstream of the shock appearing in front of the obstacle, and was determined from the experimental data reported in [1, 2]. Comparison with the experimentally determined Nu ratios for subsonic jets $Re = 3.1 \cdot 10^4$ to $5.5 \cdot 10^5$ [3, 5, 7, 8] and supersonic jets $Re = 10^7$ to $5 \cdot 10^7$ [1, 2] shows that the factor K is approximately constant and equal to 0.17 and 0.19, respectively, in the case of subsonic and supersonic jets.

Figure 1 shows experimental data taken from [1-3, 5, 7, 8] for comparison. Note that the insignificant spread in the values of K and Nu can be attributed to errors in the determination of ε.

NOTATION

y, r	are the cylindrical coordinates;
v_y, v_r	are the components of velocity;
β	is the constant in linear distribution of radial flow velocity in the neighborhood of the stagnation point;
a_*	is the speed of sound;
T	is the temperature;
q	is the heat flux;
ν	is the viscosity;
ν_ε	is the additional viscosity due to turbulent pulsations of jet;
ε	is the intensity of turbulence;
Nu, Re, Pr	is the Nusselt number, Reynolds number, Prandtl number;
M	is the Mach number;
y_0	is the distance from nozzle exit section to obstacle;
n	is the incalculability factor of jet;
d	is the diameter of nozzle, jet;
$\varkappa$	is the adiabatic exponent;
w	is the wall;
∞	is the inviscid stream;
a	is the nozzle exit section;
1	are parameters downstream of shock appearing in front of obstacle;
*	are critical parameters.

LITERATURE CITED

1. I. P. Ginzburg, I. A. Belov, V. A. Zazimko, and V. S. Terpigor'ev, Proc. III All-Union Conf. on Heat and Mass Transfer [in Russian], Vol. 1, Izd. Énergiya (1968).
2. I. A. Belov, I. P. Ginzburg, V. A. Zazimko, and V. S. Terpigor'ev, Abstracts of Reports Delivered at the III All-Union Congress on Theoretical and Applied Mechanics [in Russian], Izd. AN SSSR (1968).
3. E. H. Comfort, T. O'Connor, and L. A. Cass, Proc. Heat Transfer and Fluid Mech. Inst., Santa Clara, Calif. (1966).
4. M. C. Smith and A. M. Kueth, Physics of Fluids, 9, No. 12 (1966).
5. P. M. Brdlik and V. K. Savin, Inzh.-Fiz. Zh., 10, No. 4 (1966).
6. P. O. Davies, M. J. Fisher, and M. J. Barratt, J. Fluid Mech., 15, No. 3 (1963).
7. Huang, Teploperedacha, 85, No. 3 (1963).
8. R. Gordon and J. Cobonque, J. Internat. Heat Transfer Conf., II, New York (1961).